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EMBEDDING THRESHOLD CONCEPTS INTO HIERARCHICAL CONCEPT STRUCTURES

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INTRODUCTION

The notion of Threshold Concepts has been introduced just about a decade ago (Meyer and land 2003). Since then, a great amount of work dealing with them has been published. Even if some of those publications deal with ontological considerations, there are - from the author's point of view - still some questions which remain not clarified. For example: Is it necessary that a concept needs to fulfil all five characteristics as proposed by Meyer and Land (2003) in order to constitute a threshold concept? Or is there a smaller subset of characteristics which imply the remaining ones anyway? How are threshold concepts related to other kind of concepts, such as core concepts or fundamental ideas?

We aim to answer these questions by embedding the threshold concept into a web or hierarchical structure encompassing also other notions of concepts (e.g. Anchor Concepts). For establishing such a hierarchical structure, we apply the Formal Concept Analysis (Wille 1982) which delivers also sub-supra concept relations and attribute-implications.

DEFINITION OF THE FORMAL CONTEXT

The Formal Concept Analysis (FCA), established by Wille (1982), is a framework to describe concepts and concept hierarchies in mathematical terms, based on the application of order and lattice theory. The starting point for the FCA is the definition of the formal context. The formal context *K* is defined as a triple (*G*, *M*, *I*) with *G* as a set of objects (in German: "Gegenstände"), *M* as a set of attributes *M* (in German: "Merkmale") and *I* as a binary relation between *G* and *M*. The relation *I* connects objects and attributes, i.e. (*g*, *m*) $\in$ *I* means the object *g* has the attribute m. The formal context *K* can be best read when depicted as a cross table, with the objects in the rows, the attributes in the columns and relations between them by assigning "X" in the according cells (see Table 1). Before that, we will describe the building blocks of the formal context, i.e. the objects and their attributes.

Threshold Concepts (TCs) share the following characteristics (or attributes) as described by Meyer and Land (2003): i) *transformative*: once understood, they lead to a significant shift appears in the student`s perception of the subject, ii) *integrative*: they integrate different aspects of the subject, iii) *irreversible*: once understood, they are difficult to unlearn, iv) *bounded*: they delineate a

particular conceptual space, and finally, v) *troublesome*: they are a hard to learn or grasp for the learner.

To distinguish TCs from ***Core Concepts*** (CCs) Meyer and Land (2003) state that a core concept "…has to be understood but it does not necessarily lead to a qualitatively different view of subject matter" (p. 4). This implies that CCs do not possess the transformative characteristic. Carstensen and Bernhard (2007) use the attribute *fundamental* to describe the notion that a concept has to be understood to progress in understanding of the subject.

The notion of ***Key Concepts*** (KCs) is rarely defined by characteristics in literature but Carstensen and Bernhard (2007) suggest to "use the term key concepts for those concepts that open up the 'portal' ... (of understanding)" (p. 143). In this sense, KCs can be characterized as being *fundamental*. They have to be grasped in order to pass the 'portal'. Marsh (2009) states that "...key concepts provide us with the power to explore a variety of situations ... and to make significant connections..." (p. 9). This is in line with the *integrative* characteristic.

Mead et al. (2006) introduced ***Anchor Concepts***. They define them as either i) *integrative AND transformative* (in the same sense as described above) or ii) *foundational*, i.e. "...a critical, basic concept ... not derivable in that domain" (p. 187). We will separate this definition into two objects: for i) we will refer to the term anchor-threshold concepts (ATCs) and for ii) we will use the term anchor concepts (ACs).

Fundamental ideas have been proposed by Bruner (1960) but unfortunately he doesn´t provide an explicit definition. However, Schwill (1994) summarized the characteristics as: i) *horizontal criterion*: the concept is applicable or in different areas of the domain, ii) *vertical criterion*: it may be taught on every intellectual level. Schwill extends Bruner´s initial formulation by two additional characteristics: iii) *criterion of sense*: it embodies everyday life meaning, and finally iv) *criterion of time*: it is and will be relevant in the longer term. Schwill´s four characteristics are used by Zendler and Spannagel (2008) to define ***Central Concepts*** (CCs).

The formal context $K := (G, M, I)$ is shown in Table 1. The cells marked with an X indicate that the pair $(g, m) \in I$. The cells highlighted in grey represent binary relation directly derived from literature cited above.

Table 1: Formal Context including objects, attributes and incidents relations

	transformative	integrative	irreversible	bounded	troublesome	fundamental	foundational	horizontal criterion	vertical criterion	criterion of time	criterion of sense
Threshold Concepts	X	X	X	X	X					X	
Core Concepts				X		X				X	
Key Concepts		X		X		X				X	
Anchor-Threshold Concepts	X	X								X	

Anchor Concepts						X	X			X	
Fundamental Ideas								X	X	X	
Central Concepts								X	X	X	X

However, we assigned additional binary relations: For example, taken into account that the attribute criterion of time is quite vague (what exactly constitutes a "longer term"?), it is reasonable to assume that this attribute applies to all objects (otherwise they are not "important" building blocks of a discipline). More important, a concept which is *foundational* (i.e. axiomatic) implies the characteristic *fundamental* (i.e. they required for further progress in the discipline).

CONCEPT LATTICE

In order to create a concept lattice, for each subset A $\subseteq$ G and B $\subseteq$ M the following derivation operators need to be defined:

$$A \mapsto A' := \{m \in M \mid g\ I\ m \text{ for all } g \in A\}$$

$$B \mapsto B' := \{g \in G \mid g\ I\ m \text{ for all } m \in B\}$$

A formal concept is a pair (A, B) with the subsets $A \subseteq G$ and $B \subseteq M$ which fulfil $A = B'$ and $B' = A$. The set of objects A is called the *extension* of the formal concept; it is the set of objects of the formal concept. And B is called the *intension*, i.e. the set of attributes which apply to all objects of the extension. The ordered set of all formal concepts is called the concept lattice $\mathfrak{B}(K)$ (for theorems see Wille 2005). The concept lattice can be visualized by a labelled line diagram (see Figure 1). Every node represents a single formal concept. The *extension* of a particular formal concept is constituted the objects which can be reached by descending paths from that node. The *intension* is represented by all attributes which can be reached by an ascending path from that node.

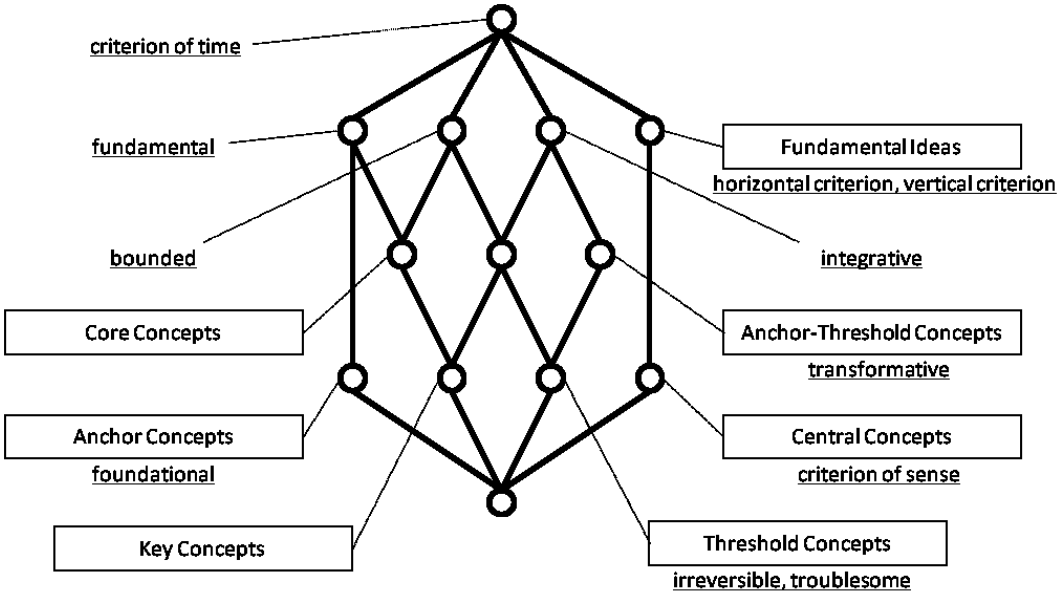

Figure 1: Concept Lattice of the Formal Context defined in Table 1

For example, the node with the label “integrative” represents a formal concept with {*Anchor-Threshold Cs*., *Key Cs*. *Threshold Cs*.} as extension and {*integrative*, *criterion of time*} as intension.

CONCLUSIONS

The concept lattice shown in Figure 1 indicates sub-supra concept relations. Supra-concepts of a particular formal concept are those which can be reached by a descending path. For example: The node labeled with “*Anchor-Threshold Concepts*” is a supra-concept of the node labeled as “*Threshold Concepts*”. ATCs possess a subset of the attributes which constitute TCs and in consequence, they are more generic. An equivalent situation occurs with the nodes labeled as “Fundamental Ideas” (as supra-concept) and “Central Concepts” (as sub-concept). In addition to this kind of relation, the concept lattice indicates attribute implications: The (distinguishing) attributes of a particular formal concept implies those attributes which can be reached by ascending paths. In other words: attributes which are possessed by a sub-concept but not by its supra-concept(s) imply the whole set of attributes constituting the supra-concept. For example, the attributes {*irreversible*, *troublesome*} imply the attributes {*transformative*, *integrative*, *bounded*, *criterion of time*}. The set {irreversible, troublesome} can be considered as the key attributes of TCs which distinguishes them from ATCs.

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